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13. SUPPLEMENTARY NOTES The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.				
14. ABSTRACT Optical lattice systems provide an ideal platform for investigating entanglement because of their unprecedented level of flexibility and dynamical control in experiments. The goal of this research project was to utilize novel superlattice geometries where each lattice site is composed of a multiple-well potential. More specifically, the goals were: (i) Prepare and characterize various initial states of a rubidium and/or potassium system experimentally and theoretically. (ii) After an instantaneous or continuous change of the system Hamiltonian, follow the entanglement dynamics of the system using various theoretical techniques and compare to experimental measurements of				
15. SUBJECT TERMS optical lattice, cold atoms				
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			19b. TELEPHONE NUMBER 509-335-2412	

Report Title

Final report for ARO grant entitled "Quantum Entanglement in Optical Lattice Systems"

ABSTRACT

Optical lattice systems provide an ideal platform for investigating entanglement because of their unprecedented level of flexibility and dynamical control in experiments. The goal of this research project was to utilize novel superlattice geometries where each lattice site is composed of a multiple-well potential. More specifically, the goals were: (i) Prepare and characterize various initial states of a rubidium and/or potassium system experimentally and theoretically. (ii) After an instantaneous or continuous change of the system Hamiltonian, follow the entanglement dynamics of the system using various theoretical techniques and compare to experimental measurements of correlation functions that quantify the entanglement. The studies aimed to provide deep insights into the emergence of entanglement and its connection to equilibration and thermalization mechanisms. A detailed understanding of these processes is essential for utilizing the full potential of entanglement and to refine quantum technology applications. As described in this and our progress reports, important results were obtained during the duration of the ARO grant.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

<u>Received</u>	<u>Paper</u>
02/18/2015 41.00	C. Hamner, Yongping Zhang, M. A. Kamehchi, Matthew J. Davis, P. Engels. Spin-orbit coupled Bose-Einstein condensates in a one-dimensional optical lattice, Phys. Rev. Lett. (accepted), (03 2015): 0. doi:
08/04/2011 4.00	Li Mao, Junren Shi, Qian Niu, Chuanwei Zhang. Superconducting Phase with a Chiral f-Wave Pairing Symmetry and Majorana Fermions Induced in a Hole-Doped Semiconductor, Physical Review Letters, (4 2011): 0. doi: 10.1103/PhysRevLett.106.157003
08/04/2011 5.00	Machi Zhang, Hsiang-hsuan Hung, Chuanwei Zhang, Congjun Wu. Quantum anomalous Hall states in the p-orbital honeycomb optical lattices, Physical Review A, (2 2011): 0. doi: 10.1103/PhysRevA.83.023615
08/04/2011 6.00	K. Daily. Few-body resonances of unequal-mass systems with infinite interspecies two-body s-wave scattering length, Physical Review A, (12 2010): 0. doi: 10.1103/PhysRevA.82.063612
08/04/2011 7.00	D. Blume, K. Daily. Breakdown of Universality for Unequal-Mass Fermi Gases with Infinite Scattering Length, Physical Review Letters, (10 2010): 0. doi: 10.1103/PhysRevLett.105.170403
08/04/2011 8.00	C. Hamner, J. Chang, P. Engels, M. Hoefer. Generation of Dark-Bright Soliton Trains in Superfluid-Superfluid Counterflow, Physical Review Letters, (2 2011): 0. doi: 10.1103/PhysRevLett.106.065302
08/04/2011 9.00	D. Blume, K.M. Daily. Trapped two-component Fermi gases with up to six particles: Energetics, structural properties, and molecular condensate fraction, Comptes Rendus Physique, (01 2011): 0. doi: 10.1016/j.crhy.2010.11.010
08/15/2012 10.00	Li Mao, Ming Gong, E. Dumitrescu, Sumanta Tewari, Chuanwei Zhang. Hole-Doped Semiconductor Nanowire on Top of an s-Wave Superconductor: A New and Experimentally Accessible System for Majorana Fermions, Physical Review Letters, (04 2012): 0. doi: 10.1103/PhysRevLett.108.177001
08/15/2012 11.00	Gang Chen, Ming Gong, Chuanwei Zhang. BCS-BEC crossover in spin-orbit-coupled two-dimensional Fermi gases, Physical Review A, (1 2012): 0. doi: 10.1103/PhysRevA.85.013601
08/15/2012 12.00	Yongping Zhang, Li Mao, Chuanwei Zhang. Mean-Field Dynamics of Spin-Orbit Coupled Bose-Einstein Condensates, Physical Review Letters, (1 2012): 0. doi: 10.1103/PhysRevLett.108.035302
08/15/2012 13.00	Ming Gong, Sumanta Tewari, Chuanwei Zhang. BCS-BEC Crossover and Topological Phase Transition in 3D Spin-Orbit Coupled Degenerate Fermi Gases, Physical Review Letters, (11 2011): 0. doi: 10.1103/PhysRevLett.107.195303
08/15/2012 14.00	K. Daily, D. Rakshit, D. Blume. Degeneracies in Trapped Two-Component Fermi Gases, Physical Review Letters, (07 2012): 0. doi: 10.1103/PhysRevLett.109.030401

- 08/15/2012 15.00 D. Rakshit, K. Daily, D. Blume. Natural and unnatural parity states of small trapped equal-mass two-component Fermi gases at unitarity and fourth-order virial coefficient, Physical Review A, (03 2012): 0. doi: 10.1103/PhysRevA.85.033634
- 08/15/2012 16.00 M. Hoefer, J. Chang, C. Hamner, P. Engels. Dark-dark solitons and modulational instability in miscible two-component Bose-Einstein condensates, Physical Review A, (10 2011): 0. doi: 10.1103/PhysRevA.84.041605
- 08/15/2012 17.00 D. Yan, J. Chang, C. Hamner, P. Kevrekidis, P. Engels, V. Achilleos, D. Frantzeskakis, R. Carretero-González, P. Schmelcher. Multiple dark-bright solitons in atomic Bose-Einstein condensates, Physical Review A, (11 2011): 0. doi: 10.1103/PhysRevA.84.053630
- 08/15/2012 18.00 D Yan, J J Chang, C Hamner, M Hoefer, P G Kevrekidis, P Engels, V Achilleos, D J Frantzeskakis, J Cuevas. Beating dark-dark solitons in Bose-Einstein condensates, Journal of Physics B: Atomic, Molecular and Optical Physics, (06 2012): 0. doi: 10.1088/0953-4075/45/11/115301

TOTAL: 16

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

<u>Received</u>	<u>Paper</u>
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TOTAL:

(c) Presentations

- 1.) P. Engels, Investigating quantum dynamics with Bose-Einstein Condensates, invited talk at the Eighth IMACS International Conference on Nonlinear Evolution Equations and Wave Phenomena: Computation and Theory, University of Georgia, Athens, GA, March 25-28, 2013
- 2.) P. Engels, New trends in BEC hydrodynamics: novel types of solitons and dispersion engineering, invited talk at AMO Physics seminar, Stony Brook University, February 4, 2013
- 3.) P. Engels, Superfluid hydrodynamics with Bose-Einstein condensates: engineering quantum dynamics, invited talk at The School of Natural Sciences physics seminar, University of Merced, California, November 30, 2012
- 4.) P. Engels, Plenary talk: Quantum hydrodynamics in dilute-gas Bose-Einstein condensates, invited plenary talk at the 14th Annual Meeting of the Northwest Section of the APS, Vancouver, BC, Canada, October 18-20, 2012
- 5.) C. Zhang, Topological Superfluids in Spin-Orbit Coupled Cold Fermi Gases: a Roadmap to Majorana Fermions, AMO Seminar, Department of Physics, Rice University September 2012, Houston, Texas
- 6.) C. Zhang, Search for Majorana fermions in spin-orbit coupled superconductors and superfluids, Physics Colloquium, Department of Physics, University of Washington, October 2012, Seattle, Washington.
- 7.) C. Zhang, Search for Majorana fermions in spin-orbit coupled degenerate Fermi gases, Condensed Matter seminar, Department of Physics, Purdue University, March 2013, West Lafayette, IN.
- 8.) C. Zhang, Search for Majorana fermions in spin-orbit coupled degenerate Fermi gases, American Physical Society March Meeting invited talk, March 2013, Baltimore, Maryland.
- 9.) D. Blume, Strongly-Correlated Two-Component Few-Fermion Systems, Invited talk at International Conference on Few-Body Physics in Cold Atomic Gases, Beijing, China; April 11-14, 2013.
- 10.) D. Blume, Pseudopotentials for Interacting Dipoles, Invited talk at Workshop entitled "Fundamental Science and Applications of Ultra-cold Polar Molecules" at the KITP, University of California, Santa Barbara, California, USA; February 6, 2013.
- 11.) D. Blume, Few-body physics with cold atoms: Techniques and results that may be of interest to nuclear physics/physicists, Invited talk during INT program on "Light Nuclei from First Principles", Seattle, USA; October 2, 2012.
- 12.) D. Blume, Universal Low-Energy Phenomena of Few-Body Systems: Application of the Stochastic Variational Approach, Invited talk at the 20th International IUPAP Conference on Few-Body Problems in Physics, Fukuoka, Japan; August 20-25, 2012)
- 13.) D. Blume, Quasi-One-Dimensional Quantum Gases: Effective Interactions, Energetics and Correlations, Atomic Physics Seminar, Stony Brook; February 25, 2013.
- 14.) D. Blume, Quasi-One-Dimensional Quantum Gases: Effective Interactions, Energetics and Correlations, Special Physics Colloquium, Purdue University, West Lafayette; February 19, 2013.
- 15.) D. Blume, Quasi-One-Dimensional Quantum Gases: Effective Interactions, Energetics and Correlations, Physics Colloquium, UMass, Boston; February 13, 2013.
- 16.) D. Blume, s-wave interacting fermions under anisotropic harmonic confinement, Seminar, Zentrum fuer Optische Quantentechnologien, Universitaet Hamburg, Germany; December 8, 2012.
- 17.) D. Blume, s-wave interacting fermions under anisotropic harmonic confinement: Dimensional crossover of energetics and virial coefficients, ITAMP topical discussion, Cambridge, Massachusetts; November 1, 2012.
- 18.) D. Blume, s-wave interacting fermions under anisotropic harmonic confinement: Dimensional crossover of energetics and virial coefficients, ABC Physics (Atomic/Bio/Condensed Matter) Seminar, University of Washington, Seattle, Washington; October 5, 2012.
- 19.) D. Blume, s-wave interacting fermions under anisotropic harmonic confinement: Dimensional crossover of energetics and virial coefficients, Atomic Physics Seminar, University of Connecticut, Storrs, Connecticut; September 24, 2012.
- 20.) D. Blume, Two-Component Fermi Gases with Unequal Masses: Three-, Four- and Many-Body Physics, Special Seminar at Tokyo University, Tokyo, Japan; August 27, 2012.

Number of Presentations: 20.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

(d) Manuscripts

<u>Received</u>	<u>Paper</u>
02/18/2015 30.00	Seyed Ebrahim Gharashi, D. Blume. Correlations of the Upper Branch of 1D Harmonically Trapped Two-Component Fermi Gases, Physical Review Letters (07 2013)
02/18/2015 40.00	A Álvarez, J Cuevas, F R Romero, C Hamner, J J Chang, P Engels, P G Kevrekidis, D J Frantzeskakis. Scattering of atomic dark-bright solitons from narrow impurities, Journal of Physics B: Atomic, Molecular and Optical Physics (03 2013)
02/18/2015 39.00	Qizhong Zhu, Chuanwei Zhang, Biao Wu. Exotic superfluidity in spin-orbit coupled Bose-Einstein condensates, EPL (Europhysics Letters) (12 2012)
02/18/2015 38.00	Chris Hamner, Chunlei Qu, Yongping Zhang, JiaJia Chang, Ming Gong, Chuanwei Zhang, Peter Engels. Dicke-type phase transition in a spin-orbit-coupled Bose-Einstein condensate, Nature Communications (06 2014)
02/18/2015 37.00	C. Hamner, Yongping Zhang, J. J. Chang, Chuanwei Zhang, P. Engels. Phase Winding a Two-Component Bose-Einstein Condensate in an Elongated Trap: Experimental Observation of Moving Magnetic Orders and Dark-Bright Solitons, Physical Review Letters (12 2013)
02/18/2015 36.00	Chunlei Qu, Chris Hamner, Ming Gong, Chuanwei Zhang, Peter Engels. Observation of Zitterbewegung in a spin-orbit-coupled Bose-Einstein condensate, Physical Review A (08 2013)
02/18/2015 35.00	Yongping Zhang, Chuanwei Zhang. Bose-Einstein condensates in spin-orbit-coupled optical lattices: Flat bands and superfluidity, Physical Review A (02 2013)
02/18/2015 34.00	Yinyin Qian, Ming Gong, Chuanwei Zhang. Many-body Landau-Zener transition in cold-atom double-well optical lattices, Physical Review A (01 2013)
02/18/2015 33.00	Ming Gong, Gang Chen, Suotang Jia, Chuanwei Zhang. Searching for Majorana Fermions in 2D Spin-Orbit Coupled Fermi Superfluids at Finite Temperature, Physical Review Letters (09 2012)
02/18/2015 32.00	Seyed Ebrahim Gharashi, K. M. Daily, D. Blume. Three s-wave-interacting fermions under anisotropic harmonic confinement: Dimensional crossover of energetics and virial coefficients, Physical Review A (10 2012)
02/18/2015 31.00	D. Rakshit, D. Blume. Hyperspherical explicitly correlated Gaussian approach for few-body systems with finite angular momentum, Physical Review A (12 2012)
02/18/2015 29.00	Seyed Ebrahim Gharashi, X. Y. Yin, D. Blume. Molecular branch of a small highly elongated Fermi gas with an impurity: Full three-dimensional versus effective one-dimensional description, Physical Review A (02 2014)
03/15/2011 1.00	Y. Qian, M. Gong, C. Zhang. Quantum Transport of Bosonic Cold Atoms in Double Well Optical Lattices, (03 2011)

03/15/2011 2.00 Y. Zhang, L. Mao, C. Zhang. Gross-Pitaevskii Equation and Center of Mass Motion in Spin-Orbit Coupled Bose-Einstein Condensates, (03 2011)

05/18/2011 3.00 M. Gong, S. Tewari, C. Zhang. BCS-BEC Crossover and Topological Phase Transition in Spin-Orbit Coupled Degenerate Fermi Gases, (05 2011)

08/15/2012 22.00 Ming Gong, Yinyin Qian, V. W. Scarola, Chuanwei Zhang. Dzyaloshinskii-Moriya Interaction and Spiral Order in Spin-orbit Coupled Optical Lattices, arXiv:1205.6211v1 (05 2012)

08/15/2012 20.00 Ming Gong, Gang Chen, Suotang Jia, Chuanwei Zhang. Searching for Majorana Fermions in 2D Spin-orbit Coupled Fermi Superfluids at Finite Temperature, ()

08/15/2012 21.00 Li Mao, Sumanta Tewari, Chuanwei Zhang, Ming Gong. Majorana Fermions Under Stress, arXiv:1205.6209v1 (05 2012)

08/15/2012 23.00 Yongping Zhang, Chuanwei Zhang. BEC in Spin-Orbit Coupled Optical Lattices: Flat Bands and Superfluidity, arXiv:1203.2389v1 (03 2012)

08/15/2012 24.00 Yongping Zhang, Gang Chen, Chuanwei Zhang. Tunable Spin-orbit Coupling and Quantum Phase Transition in a Trapped Bose-Einstein Condensate, arXiv:1111.4778v1 (11 2011)

08/15/2012 25.00 Yinyin Qian, Ming Gong, Chuanwei Zhang. Many-body Landau-Zener Transition in Cold Atom Double Well Optical Lattices, arXiv:1110.1653v2 (11 2011)

08/15/2012 26.00 Qizhong Zhu, Chuanwei Zhang, Biao Wu. Exotic superfluidity in spin-orbit coupled Bose-Einstein condensates, arXiv:1109.5811v2 (09 2011)

08/15/2012 27.00 Chunlei Qu, Yongping Zhang, Li Mao, Chuanwei Zhang. Signature of Majorana Fermions in Charge Transport in Semiconductor Nanowires, arXiv:1109.4108v2 (09 2011)

TOTAL: 23

Number of Manuscripts:

Books

Received

Book

TOTAL:

Received

Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

Graduate Students

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	Discipline
S. E. Gharashi	1.00	
X. Y. Yin	0.50	
FTE Equivalent:	1.50	
Total Number:	2	

Names of Post Doctorates

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
R. Chu	0.50
FTE Equivalent:	0.50
Total Number:	1

Names of Faculty Supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	National Academy Member
D. Blume	0.00	
P. Engels	0.10	
C. Zhang	0.00	
FTE Equivalent:	0.10	
Total Number:	3	

Names of Under Graduate students supported

NAME

PERCENT SUPPORTED

FTE Equivalent:

Total Number:

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:..... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):..... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:..... 0.00

Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PHDs

NAME

Total Number:

Names of other research staff

NAME

PERCENT SUPPORTED

FTE Equivalent:

Total Number:

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Technology Transfer

nothing to report

Final scientific report for grant entitled “Quantum Entanglement in Optical Lattice systems”

PIs: Doerte Blume, Peter Engels, and Chuanwei Zhang

Funding Period: May 16, 2009 – May 15, 2013

Optical lattice systems provide an ideal platform for investigating entanglement because of their unprecedented level of flexibility and dynamical control in experiments. The goal of this research project was to utilize novel superlattice geometries where each lattice site is composed of a multiple-well potential. More specifically, the goals were: (i) Prepare and characterize various initial states of a rubidium and/or potassium system experimentally and theoretically. (ii) After an instantaneous or continuous change of the system Hamiltonian, follow the entanglement dynamics of the system using various theoretical techniques and compare to experimental measurements of correlation functions that quantify the entanglement. The studies aimed to provide deep insights into the emergence of entanglement and its connection to equilibration and thermalization mechanisms. A detailed understanding of these processes is essential for utilizing the full potential of entanglement and to refine quantum technology applications. As detailed below, important results were obtained during the fourth funding period of this project. The accomplishments during the first three funding periods were summarized in the scientific progress reports submitted in summer 2010, summer 2011 and summer 2012, and are not reiterated here.

During the fourth funding period (August 2012 – May 2013) our investigations focused on one-dimensional atomic gases and on spin-orbit coupled Bose-Einstein condensates (BECs). On the experimental side, the superlattice setup was extended to a Raman setup that realizes artificial gauge fields and spin-orbit coupling. These extensions of the experiments conducted by the Engels group were motivated by the theoretical investigations by Zhang’s group. The Raman lasers are arranged such that a Raman transition between two hyperfine states is accompanied by a change of momentum. This leads to an effective spin-orbit coupling. Since the Raman coupling strength and the detuning from the Raman resonance can be independently adjusted in our experiment, this provides a very flexible platform for engineering interesting dispersion relations. From a technical point of view, the setup required a very careful stabilization of the magnetic fields. After the successful implementation of this scheme, we conducted a series of experiments investigating the peculiar features of spin-orbit coupled BECs, including studies of quench dynamics, Dicke-type physics, and spin-orbit coupled lattice systems. Our experiments were closely accompanied by theoretical studies conducted by Zhang and collaborators.

Theoretical efforts by Blume’s group focused on understanding correlations of one- and quasi-one-dimensional Fermi gases. These studies were motivated by experimental efforts by Jochim’s group in Heidelberg. In these experiments, one-dimensional atomic Fermi samples consisting of two or more particles can be prepared deterministically. Our theoretical efforts concentrated on estimating the relevance of the tight confinement direction, on quantifying the validity regime of the effective one-dimensional

Hamiltonian in the molecular regime, and on characterizing the correlations of the upper branch.

In the following, we summarize selected highlights from the studies conducted during the fourth funding period.

1.) Quench dynamics and Zitterbewegung:

As a first application of our spin-orbit coupled system, we conducted a detailed analysis of the effects of quantum quenches generated by rapid changes of a parameter in the Hamiltonian governing the many-body dynamics. This led us to the observation of Zitterbewegung (trembling motion), i.e. a simultaneous oscillation of velocity (or position) and spin. The Zitterbewegung oscillation was first predicted by Schroedinger in 1930 for relativistic Dirac electrons where it arises from the interference between particle and hole components of Dirac spinors. Although fundamentally important, the Zitterbewegung oscillation is difficult to observe in “real” particles. In the past eight decades, analogs of the Zitterbewegung oscillation have been predicted to exist in various physical systems, ranging from solid state (e.g., semiconductor quantum wells) to trapped cold atoms, but experimentally, a Zitterbewegung analog had been observed only in one previous system using trapped ions as a quantum emulator of the Dirac equation.

The experiments conducted by the Engels group have clearly revealed the Zitterbewegung in a spin-orbit coupled BEC following a quantum quench. Furthermore, we demonstrated how a sequence of two quenches can be used to populate the higher spin-orbit band. The experimental results were complemented by a thorough numerical and analytic investigations.

Our work has showcased the exceptional flexibility that cold atoms provide for the study of quantum spin dynamics in spin-orbit-coupled superfluids and has laid the foundation for further studies of spin decoherence, upper band dynamics, etc. The experimental and theoretical results of these studies are published as a Rapid Communication in Physical Review A (PRA 88, 021604(R) (2013)).

2.) Dicke-type physics:

Following our investigation of quantum quenches, we focused on the ground state properties of a spin-orbit coupled BEC and showed that the system can be mapped to the well-known Dicke model in quantum optics, which describes the interactions between an ensemble of atoms and an optical field. A central prediction of the Dicke model is a quantum phase transition between a superradiant phase and a normal phase. In our experiments, conducted by the Engels group, as well as in our numerics, performed by the Zhang group, we clearly detected this transition and demonstrated the applicability and usefulness of mapping the spin-orbit BEC to a Dicke-type system. We analyzed various physical quantities across the phase transition including the spin polarization, the relative occupation of the nearly degenerate single particle states, the quantity analogous to the photon field occupation, and the period of a collective oscillation (quadrupole

mode). The applicability of the Dicke model to spin-orbit coupled BECs offers a new viewpoint on spin-orbit coupled BECs and may lead to interesting applications in quantum optics and quantum information science. The results of these studies are published as an article in Nature Communications (doi:10.1038/ncomms5023).

3.) Spin-orbit lattice systems:

Spin-orbit coupling and optical lattices are two techniques that directly modify the dispersion relation of atoms. Intriguing consequences arise when both techniques are simultaneously applied to a BEC. In a seminal investigation, the Zhang group theoretically analyzed a spin-orbit coupled lattice system, leading to exciting predictions such as the existence of flat bands (Physical Review A 87, 023611 (2013)). This motivated experimental studies by the Engels group. In the experiments, the modulational instability of a spin-orbit coupled BEC confined in an optical lattice was exploited to measure the position of band gaps in the band structure. The positions of the band gaps were shown to be in excellent agreement with predictions made by Yongping Zhang, a former member of Zhang's group, who had developed an effective band structure picture for spin-orbit coupled lattice systems. This study resulted in a clear and insightful physical picture for the dispersion of spin-orbit coupled lattice systems (arXiv:1405.4048, accepted for publication in Physical Review Letters).

4.) Dynamics in two-component systems:

We also continued our line of research investigating dynamics in two-component systems. Of particular note is our study of inhomogeneous phase winding in BECs. The phase winding was induced by subjecting an elongated BEC to extended Rabi cycling in the presence of a magnetic gradient. The gradient affected the Rabi cycling rate, leading to a phase winding along the long axis of the cigar-shaped BEC. While the single-particle dynamics could be explained by mapping the system to a two-component Bose-Hubbard model, nonlinearities due to the interatomic repulsion led to new effects observed in the experiments: In the presence of a linear magnetic field gradient, a qualitatively stable moving magnetic order that is similar to antiferromagnetic order was observed after critical winding was achieved. We also demonstrated how the phase winding can be used to generate copious dark-bright solitons in a two-component BEC, opening the door for new experimental studies of these nonlinear features. The experiments, conducted by the Engels group, were accompanied by numerical and analytic theoretical studies performed by the Zhang group. This work resulted in a combined experimental / theoretical publication in Physical Review Letters (Phys. Rev. Lett. 111, 264101(2013)).

5.) Three s-wave-interacting fermions under anisotropic harmonic confinement: Dimensional crossover of energetics and virial coefficients:

We obtained highly accurate solutions of the Schrödinger equation for three fermions in two different spin states with zero-range s-wave interactions under harmonic confinement. Our approach is applicable to spherically symmetric, strictly two-dimensional, strictly one-dimensional, cigar-shaped, and pancake-shaped traps and has

since been used by other groups. We characterized the transition from quasi-one-dimensional to strictly one-dimensional and from quasi-two-dimensional to strictly two-dimensional geometries. We determined and interpreted the eigenenergies of the system as a function of the trap geometry and the strength of the zero-range interactions. The eigenenergies were then used to investigate the dependence of the second- and third-order virial coefficients, which play an important role in the virial expansion of the thermodynamic potential, on the geometry of the trap. We found that the second- and third-order virial coefficients for anisotropic confinement geometries are, for experimentally relevant temperatures, very well approximated by those for the spherically symmetric confinement for all s-wave scattering lengths. Our results are summarized in Phys. Rev. A 86, 042702 (2012).

6.) Molecular branch of a small highly elongated Fermi gas with an impurity: Full three-dimensional versus effective one-dimensional description:

Extending the work discussed in 5.), we considered an impurity immersed in a small Fermi gas under highly elongated harmonic confinement. The impurity was assumed to interact with the atoms of the Fermi gas through an isotropic short-range potential with three-dimensional free-space s-wave scattering length. We investigated the energies of the molecular branch, i.e., the energies of the state that corresponds to a gas consisting of a weakly bound diatomic molecule and “unpaired” atoms, as a function of the s-wave scattering length and the ratio between the angular trapping frequencies in the tight and weak confinement directions. The energies obtained from our three-dimensional description that accounts for the dynamics in the weak and tight confinement directions were compared with those obtained within an effective one-dimensional framework, which accounts for the dynamics in the tight confinement direction via a renormalized one-dimensional coupling constant. Our theoretical results were related to recent experimental measurements by the Jochim group in Heidelberg. Specifically, our theoretical results were used extensively in the Science paper by the Jochim group (Science 342, 6157 (2013)). Our results are summarized in Phys. Rev. A 89, 023603 (2014).

7.) Correlations of the Upper Branch of 1D Harmonically Trapped Two-Component Fermi Gases:

We obtained highly accurate energy spectra and eigenfunctions of small strictly 1D harmonically trapped two-component Fermi gases with interspecies delta-function interactions, and analyzed the correlations of the so-called upper branch (i.e., the branch that describes a repulsive Fermi gas consisting of atoms but no molecules) for positive and negative coupling constants. Systems with up to four particles were considered. Changes of the two-body correlations as a function of the interspecies coupling strength reflect the competition of the interspecies interaction and the effective repulsion due to the Pauli exclusion principle, and were interpreted as a few-body analog of a transition from a nonmagnetic to a magnetic phase. Moreover, we showed that the eigenstate of the infinitely strongly interacting two-component system with more than two particles, which is reached experimentally by adiabatically changing the system parameters, does not, as

previously proposed, coincide with the wave function obtained by applying a generalized Fermi-Fermi mapping function to the eigenfunction of the non-interacting single-component Fermi gas. These results are summarized in Phys. Rev. Lett. 111, 045302 (2013) and have triggered a great deal of theoretical activity by other groups.

During the period August 2012-May 2013, Ruilin Chu (postdoc in the Zhang group), Ebrahim Gharashi (graduate student in the Blume group) and Xiangyu Yin (graduate student in the Blume group) were fully or partially supported by the ARO grant.

Ruilin Chu: supported December 2012 – May 2013

Ebrahim Gharashi: supported August 2012 – May 2013

Xiangyu Yin: supported January 2013 – May 2013

JiaJia Chang, who was supported by the ARO grant during the earlier funding periods, graduated with a Ph.D. in August 2013. Ebrahim Gharashi and Xiangyu Yin are slated to graduate in Spring and Summer 2015, respectively.

The PIs gave 20 invited talks that acknowledged ARO support during the period August 2012-May 2013. In addition, group members presented several contributed talks and posters.

Invited talks:

- 1.) P. Engels, Investigating quantum dynamics with Bose-Einstein Condensates, invited talk at the Eighth IMACS International Conference on Nonlinear Evolution Equations and Wave Phenomena: Computation and Theory, University of Georgia, Athens, GA, March 25-28, 2013
- 2.) P. Engels, New trends in BEC hydrodynamics: novel types of solitons and dispersion engineering, invited talk at AMO Physics seminar, Stony Brook University, February 4, 2013
- 3.) P. Engels, Superfluid hydrodynamics with Bose-Einstein condensates: engineering quantum dynamics, invited talk at The School of Natural Sciences physics seminar, University of Merced, California, November 30, 2012
- 4.) P. Engels, Plenary talk: Quantum hydrodynamics in dilute-gas Bose-Einstein condensates, invited plenary talk at the 14th Annual Meeting of the Northwest Section of the APS, Vancouver, BC, Canada, October 18-20, 2012
- 5.) C. Zhang, *Topological Superfluids in Spin-Orbit Coupled Cold Fermi Gases: a Roadmap to Majorana Fermions*, AMO Seminar, Department of Physics, Rice University September 2012, Houston, Texas

- 6.) C. Zhang, *Search for Majorana fermions in spin-orbit coupled superconductors and superfluids*, Physics Colloquium, Department of Physics, University of Washington, October 2012, Seattle, Washington.
- 7.) C. Zhang, *Search for Majorana fermions in spin-orbit coupled degenerate Fermi gases*, Condensed Matter seminar, Department of Physics, Purdue University, March 2013, West Lafayette, IN.
- 8.) C. Zhang, *Search for Majorana fermions in spin-orbit coupled degenerate Fermi gases*, American Physical Society March Meeting invited talk, March 2013, Baltimore, Maryland.
- 9.) D. Blume, Strongly-Correlated Two-Component Few-Fermion Systems, Invited talk at International Conference on Few-Body Physics in Cold Atomic Gases, Beijing, China; April 11-14, 2013.
- 10.) D. Blume, Pseudopotentials for Interacting Dipoles, Invited talk at Workshop entitled "Fundamental Science and Applications of Ultra-cold Polar Molecules" at the KITP, University of California, Santa Barbara, California, USA; February 6, 2013.
- 11.) D. Blume, Few-body physics with cold atoms: Techniques and results that may be of interest to nuclear physics/physicists, Invited talk during INT program on "Light Nuclei from First Principles", Seattle, USA; October 2, 2012.
- 12.) D. Blume, Universal Low-Energy Phenomena of Few-Body Systems: Application of the Stochastic Variational Approach, Invited talk at the 20th International IUPAP Conference on Few-Body Problems in Physics, Fukuoka, Japan; August 20-25, 2012)
- 13.) D. Blume, Quasi-One-Dimensional Quantum Gases: Effective Interactions, Energetics and Correlations, Atomic Physics Seminar, Stony Brook; February 25, 2013.
- 14.) D. Blume, Quasi-One-Dimensional Quantum Gases: Effective Interactions, Energetics and Correlations, Special Physics Colloquium, Purdue University, West Lafayette; February 19, 2013.
- 15.) D. Blume, Quasi-One-Dimensional Quantum Gases: Effective Interactions, Energetics and Correlations, Physics Colloquium, UMass, Boston; February 13, 2013.
- 16.) D. Blume, s-wave interacting fermions under anisotropic harmonic confinement, Seminar, Zentrum fuer Optische Quantentechnologien, Universitaet Hamburg, Germany; December 8, 2012.
- 17.) D. Blume, s-wave interacting fermions under anisotropic harmonic confinement: Dimensional crossover of energetics and virial coefficients, ITAMP topical discussion, Cambridge, Massachusetts; November 1, 2012.

18.) D. Blume, s-wave interacting fermions under anisotropic harmonic confinement: Dimensional crossover of energetics and virial coefficients, ABC Physics (Atomic/Bio/Condensed Matter) Seminar, University of Washington, Seattle, Washington; October 5, 2012.

19.) D. Blume, s-wave interacting fermions under anisotropic harmonic confinement: Dimensional crossover of energetics and virial coefficients, Atomic Physics Seminar, University of Connecticut, Storrs, Connecticut; September 24, 2012.

20.) D. Blume, Two-Component Fermi Gases with Unequal Masses: Three-, Four- and Many-Body Physics, Special Seminar at Tokyo University, Tokyo, Japan; August 27, 2012.

Papers that resulted from the ARO grant during the funding period August 2012-May 2013:

1. C. Hamner, C. Qu, Y. Zhang, J. J. Chang, M. Gong, C. Zhang, and P. Engels, Dicke-type phase transition in a spin-orbit coupled Bose-Einstein condensate, Nature Communications 5, 4023 (2014). DOI: 10.1038/ncomms5023

2. C. Hamner, Y. Zhang, J.J. Chang, C. Zhang, P. Engels, Phase Winding a Two-Component BEC in an Elongated Trap: Experimental Observation of Moving Magnetic Orders and Dark-bright Solitons, Phys. Rev. Lett. 111, 264101 (2013). DOI: 10.1103/PhysRevLett.111.264101

3. C. Qu, C. Hamner, M. Gong, C. Zhang, P. Engels, Observation of Zitterbewegung in a spin-orbit-coupled Bose-Einstein condensate, Phys. Rev. A 88, 021604(R) (2013). DOI: 10.1103/PhysRevA.88.021604

4. Y. Zhang, and C. Zhang, Bose-Einstein Condensates in Spin-Orbit Coupled Optical Lattices: Flat Bands and Superfluidity, Physical Review A 87, 023611 (2013). DOI: 10.1103/PhysRevA.87.023611

5. Y. Qian, M. Gong, and C. Zhang, Many-body Landau-Zener Transition in Cold Atom Double Well Optical Lattices, Physical Review A 87, 013636 (2013). DOI: 10.1103/PhysRevA.87.013636

6. Q. Zhu, C. Zhang, and B. Wu, Exotic Superfluidity in Spin-orbit Coupled Bose-Einstein Condensates, Europhysics Letters 100, 50003 (2012). DOI: 10.1209/0295-5075/100/50003

7. M. Gong, G. Chen, S. Jia, and C. Zhang, Searching for Majorana Fermions in 2D Spin-orbit Coupled Fermi Superfluids at Finite Temperature, Phys. Rev. Lett. 109, 105302 (2012). DOI: 10.1103/PhysRevLett.109.105302

8. C. Hamner, Y. Zhang, M. A. Kamehchi, M. Davis, and P. Engels, Spin-orbit coupled Bose-Einstein condensates in a one-dimensional optical lattice, arXiv:1405.4048, accepted for publication in Phys. Rev. Lett.
9. A. Álvarez, J. Cuevas, F. R. Romero, C. Hamner, J. J. Chang, P. Engels, P. G. Kevrekidis, and D. J. Frantzeskakis, Scattering of atomic dark-bright solitons from narrow impurities, J. Phys. B: At. Mol. Opt. Phys. **46**, 065302 (2013). DOI:10.1088/0953-4075/46/6/065302
10. S. E. Gharashi, X. Y. Yin, and D. Blume, Molecular branch of a small elongated Fermi gas with an impurity: Full three-dimensional versus effective one-dimensional description, Phys. Rev. A 89, 023603 (2014). DOI: <http://dx.doi.org/10.1103/PhysRevA.89.023603>
11. S. E. Gharashi and D. Blume, Correlations of the Upper Branch of 1D Harmonically Trapped Two-Component Fermi Gases, Phys. Rev. Lett. 111, 045302 (2013). DOI: <http://dx.doi.org/10.1103/PhysRevLett.111.045302>
12. D. Rakshit and D. Blume, Hyperspherical explicitly correlated Gaussian approach for few-body systems with finite angular momentum, Phys. Rev. A 86, 062513 (2012). DOI: <http://dx.doi.org/10.1103/PhysRevA.86.062513>
13. S. E. Gharashi, K. M. Daily, and D. Blume, Three s-wave interacting fermions under anisotropic harmonic confinement: Dimensional crossover of energetics and virial coefficients, Phys. Rev. A 86, 042702 (2012). DOI: <http://dx.doi.org/10.1103/PhysRevA.86.042702>